# Package multivator applied to modular systems such as **CIAS**

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### Abstract

This vignette shows how the **multivator** package may be used to analyze computer model systems such as CIAS, which comprise interchangeable modules. If the different modules require different input parameters, then it is possible to use specify a reasonable mean and covariance structure using the package, and a short example, using synthetic data, is given here.

*Keywords*: Emulator, multivariate emulator, BACCO, CIAS.

## 1. Introduction

CIAS (Warren *et al.* 2008) is a system for analyzing climate change in an interdisciplinary context. It has a modular architecture, one benefit of which is that a module may be replaced with another compatible module and results compared.

In CIAS, the modules address various aspects of the global climate system and examples include E3MG which models the global economy, MAGICC which models the physical global climate, and ICLIPS which models the impacts of climate change. The modules exchange information at runtime using the BFG protocol.

One feature of CIAS is that it is possible to replace any module with another functionally equivalent one. Multivariate emulation is useful when considering the behaviour of CIAS used in this way. If one has $p$ different interchangeable modules, then the output of CIAS is a $p$-variate random variable that may be analyzed using the **multivator** package.

If the interchangeable modules have the same input parameters, then analysis of the type presented in the main vignette is appropriate. If, as is usually the case, the modules have different or incomparable input parameters (possibly with parameter spaces of different dimensionality) then a slight modification is necessary.

## 2. The package in use

In this vignette, I analyze a small synthetic dataset of the type arising when CIAS is used. Consider the following multivariate design matrix:

```
> cias_mdm
```

|  | cias1 | cias2 | A_p1 | B_p1 | B_p2 | C_p1 |
|---|---|---|---|---|---|---|
| module_A_run1 | 0.643 | 0.214 | 0.262 | 0 | 0 | 0 |
| module_A_run2 | 0.167 | 0.405 | 0.357 | 0 | 0 | 0 |
| module_A_run3 | 0.310 | 0.643 | 0.548 | 0 | 0 | 0 |
| module_A_run4 | 0.024 | 0.976 | 0.976 | 0 | 0 | 0 |
| module_A_run5 | 0.071 | 0.833 | 0.310 | 0 | 0 | 0 |
| module_A_run6 | 0.595 | 0.071 | 0.119 | 0 | 0 | 0 |
| module_A_run7 | 0.881 | 0.452 | 0.452 | 0 | 0 | 0 |
| module_B_run1 | 0.500 | 0.881 | 0 | 0.929 | 0.595 | 0 |
| module_B_run2 | 0.786 | 0.929 | 0 | 0.786 | 0.071 | 0 |
| module_B_run3 | 0.976 | 0.548 | 0 | 0.548 | 0.976 | 0 |
| module_B_run4 | 0.119 | 0.024 | 0 | 0.262 | 0.024 | 0 |
| module_B_run5 | 0.833 | 0.167 | 0 | 0.024 | 0.929 | 0 |
| module_B_run6 | 0.214 | 0.119 | 0 | 0.214 | 0.357 | 0 |
| module_B_run7 | 0.405 | 0.262 | 0 | 0.690 | 0.643 | 0 |
| module_C_run1 | 0.738 | 0.738 | 0 | 0 | 0 | 0.786 |
| module_C_run2 | 0.262 | 0.310 | 0 | 0 | 0 | 0.357 |
| module_C_run3 | 0.690 | 0.690 | 0 | 0 | 0 | 0.310 |
| module_C_run4 | 0.548 | 0.786 | 0 | 0 | 0 | 0.976 |
| module_C_run5 | 0.452 | 0.357 | 0 | 0 | 0 | 0.452 |
| module_C_run6 | 0.929 | 0.595 | 0 | 0 | 0 | 0.595 |
| module_C_run7 | 0.357 | 0.500 | 0 | 0 | 0 | 0.071 |

Each row corresponds to a run of CIAS, with one of three modules, A, B, or C. Rows 1-7 used module A, rows 8-14 used module B, and rows 15-21 used module C. Parameters are indicated with column headings; thus CIAS itself has two parameters cias1 and cias2. Module A has one parameter (A_p1); B has two (B_p1 and B_p2) and module C has one (C_p1).

Now consider the hyperparameter object cias_mhp:

```
> summary(cias_mhp)


overall covariance matrix M:

  A B C
A 2 1 1
B 1 2 1
C 1 1 2



each B matrix is diagonal, with entries


$A
cias1 cias2  A_p1  B_p1  B_p2  C_p1
    1     1     1     1     1     1


$B
```

```
cias1 cias2  A_p1  B_p1  B_p2  C_p1
    1     1     1     1     1     1
```

```
$C
cias1 cias2  A_p1  B_p1  B_p2  C_p1
    1     1     1     1     1     1
```

The following `LoF` object

```
> cias_LoF
```

```
$A
function(x){ c(const=1,x[1:2],x[3  ]) }
```

```
$B
function(x){ c(const=1,x[1:2],x[4:5]) }
```

```
$C
function(x){ c(const=1,x[1:2],x[6  ]) }
```

specifies a set of basis functions that are appropriate to the multivariate design matrix `cias_mdm`. It is now possible to create some synthetic data from this system:

```
> cias_obs <- obs_maker(cias_mdm, cias_mhp, cias_LoF, cias_beta)
> cias_expt <- experiment(cias_mdm , cias_obs)
```

and so `cias_expt` is an object of class `experiment`, containing a multivariate design matrix and observations.

Now consider the following design matrix:

```
> cias_unknown
```

```
    cias1 cias2 A_p1 B_p1 B_p2 C_p1 type
m.A   0.5   0.5    0    0    0    0    A
m.B   0.5   0.5    0    0    0    0    B
m.C   0.5   0.5    0    0    0    0    C
```

The three rows corresponding to an evaluation of the three modules: the CIAS parameters are 0.5, and the module parameters take the notional value of zero.

```
> multem(cias_unknown, cias_expt, cias_mhp, cias_LoF, give=TRUE)
```

```
$mstar
      m.A        m.B        m.C
 5.431262   9.833946  22.378877
```

```
$cstar
          m.A         m.B         m.C
m.A 0.44060903  0.03343105  0.01131915
m.B 0.03343105  1.41995186 -0.01392041
m.C 0.01131915 -0.01392041  0.04724244
```

# 3. Discussion

The **multivator** package has been used to assess a modular systems such as CIAS, which has the facility to replace an individual module with another performing the same task, but possibly using a different modelling approach.

The system adopted here has some similarities to that of Kennedy and O'Hagan (2000), who gave a way of dealing with a hierarchy of models using a sequential system of basis functions and fitting a Gaussian process to the difference between successive levels in the hierarchy.

# References

Kennedy MC, O'Hagan A (2000). "Predicting the Output From a Complex Computer Code When Fast Approximations Are Available." *Biometrika*, **87**(1), 1–13.

Warren R, *et al.* (2008). "Development and Illustrative Outputs Of The Community Integrated Assessment System (CIAS), a Multi-Institutional Modular Integrated Assessment Approach for Modelling Climate Change." *Environmental Modelling and Software*, **23**, 592–610.

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